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MATDIP301

Third Semester B.E Degree Examination, June/July 2017
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Express : $\frac{1}{(2+i)^2}-\frac{1}{(2-i)^2}$ in the form of a + i b. **(07 Marks)**

b. Find the modulus and amplitude of the complex number $1-\cos\alpha+i\sin\alpha$. **(06 Marks)**

c. Express the complex number $\sqrt{3}+i$ in the polar form. **(07 Marks)**

2 a. Find the n^{th} derivative of log (ax + b). **(07 Marks)**

b. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$. **(06 Marks)**

c. If $y = \sin^{-1} x$, prove that : $(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-n^2y_n=0$. **(07 Marks)**

3 a. Using Taylor's theorem, expand sin x in power of $(x - \pi/2)$. **(07 Marks)**

b. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$ up to the term containing x^4. **(06 Marks)**

c. State and prove Euler's theorem. **(07 Marks)**

4 a. Find the total derivative of $z = xy^2 + x^2y$ where x = at, y = 2at, and also verify the result by direct substitution. **(07 Marks)**

b. If $u = f(y - z, z - x, x - y)$ prove that : $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. **(06 Marks)**

c. if $x = u(1 - v)$ and $y = uv$, find $J=\frac{\partial(x,y)}{\partial(u,v)}$ and $J'=\frac{\partial(u,v)}{\partial(x,y)}$ and also verify $J\cdot J' = 1$. **(07 Marks)**

5 a. Obtain the reduction formula for $\int \cos^n x\cdot dx$. **(07 Marks)**

b. Evaluate : $\int_0^2 \frac{x^4}{\sqrt{4-x^2}}\cdot dx$. **(06 Marks)**

c. Evaluate : $\int_1^2\int_1^3 xy^2\,dx\,dy$. **(07 Marks)**

6 a. Evaluate : $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} x\,y\,z\,dz\,dy\,dx$. **(07 Marks)**

b. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. **(06 Marks)**

c. Prove that $\beta(m,n)=\frac{\Gamma_m\Gamma_n}{\Gamma(m+n)}$. **(07 Marks)**

7 a. Solve : $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$. **(07 Marks)**

b. Solve $x^2y\,dx - (x^3 + y^3)\,dy = 0$. **(06 Marks)**

c. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(07 Marks)**

8 a. Solve : $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = 0$. **(05 Marks)**

b. Solve $\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 9y = 3e^{-4x}$. **(05 Marks)**

c. Solve : $y'' + 2y' + y = e^{-x} + \cos 2x$. **(05 Marks)**

d. Solve : $\frac{d^2y}{dx^2} - 4y = x\sin 2x$. **(05 Marks)**

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